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$$2e=\sum_{k=1}^{k=\infty}\frac{k^2}{k!},\ 5e=\sum_{k=1}^{k=\infty}\frac{k^3}{k!},\ 15e=\sum_{k=1}^{k=\infty}\frac{k^4}{k!},\ 52e=\sum_{k=1}^{k=\infty}\frac{k^5}{k!},\ 203e=\sum_{k=1}^{k=\infty}\frac{k^6}{k!},\ 877e=\sum_{k=1}^{k=\infty}\frac{k^7}{k!},$$

$$4140e=\sum_{k=1}^{k=\infty}\frac{k^8}{k!},\ 21147e=\sum_{k=1}^{k=\infty}\frac{k^9}{k!},\ 115975e=\sum_{k=1}^{k=\infty}\frac{k^{10}}{k!}.$$

Also solved by *G. B. M. ZERR.*

GEOMETRY.

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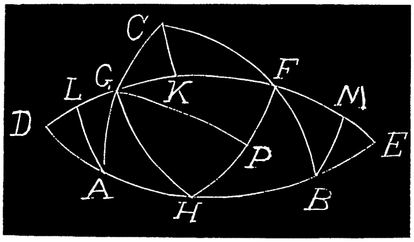
160. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let GFH be the spherical triangle formed by joining the mid-points of the sides of the spherical triangle ABC; E the spherical excess of ABC; β, p the base and altitude of GFH. Prove $\sin\frac{1}{2}E=\sin\beta\sin p$.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let $BA=c$, $GF=\gamma$, $FH=\beta$, $GH=\delta$, $GP=p$.

Draw AL, BM, CK perpendicular to $DGFC$. Now $DABE=DGFE=\pi$.



$AD=BE=\frac{1}{2}(\pi-c)$, $DL=ME$, $LG=CK$, $KF=FM$.

$\therefore 2(DL+GK+KF)=\pi$. Also $2GK+2KF=2\gamma$.

$\therefore 2DL+2\gamma=\pi$, or $DL=\frac{1}{2}\pi-\gamma$.

$\angle DAL=\angle EBM$, $\angle LAG=\angle GCK$, $\angle KCF=\angle FBM$.

$\therefore 2\angle DAL+C+A+B=2\pi$.

$\therefore \angle DAL=\pi-\frac{1}{2}(A+B+C)=\pi-s=\frac{1}{2}(\pi-E)$.

$\cos DAL=\sin D\cos DL$. $\cos\frac{1}{2}(\pi-E)=\sin D\cos(\frac{1}{2}\pi-\gamma)$.

$\therefore \sin\frac{1}{2}E=\sin D\sin\gamma$.

Now $\sin D:\sin DFH=\sin\beta:\sin DH$. But $DH=\frac{1}{2}\pi$.

$\therefore \sin D=\sin\beta\sin DFH$. But $\sin p=\sin\gamma\sin DFH$.

$\therefore \sin D=(\sin\beta\sin p)/\sin\gamma$.

$\therefore \sin\frac{1}{2}E=\sin\beta\sin p$.

Also solved by *J. SCHEFFER* and *L. C. WALKER.*

161. Proposed by MARCUS BAKER, U. S. Coast and Geodetic Survey Office, Washington, D. C.

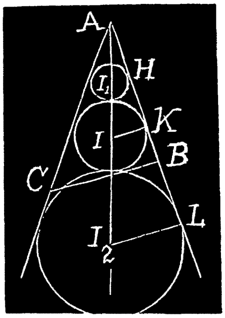
A circle, radius r, is inscribed in a triangle ABC. In the angles A, B, and C are inscribed circles each touching two sides and the inscribed circle. There are six such circles. The first group of three have their centers between the in-centers and the vertices, and the second group of three does not. Let r_a, r_b, r_c denote the radii of the first group. Then this well known relation holds: $r=\sqrt{(r_ar_b)}+\sqrt{(r_br_c)}+\sqrt{(r_cr_a)}$. Let R_a, R_b, R_c denote the radii of the second group. Then this relation holds:

$$\frac{1}{r}=\frac{1}{\sqrt{(R_aR_b)}}+\frac{1}{\sqrt{(R_bR_c)}}+\frac{1}{\sqrt{(R_cR_a)}}.$$

Required proof.

I. Solution by LON C. WALKER, A. M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Let ABC be a triangle, I the center of the in-circle with radius r. The circles with radii r_a, R_a will have centers I_1, I_2, and points of contact with the in-circle on the straight line AI. Draw the radii I_1H, IK, I_2L. The length of IA is



$$r+r_a+r_a\text{cosec}\tfrac{1}{2}A=r\text{cosec}\tfrac{1}{2}A \text{ ; thence}$$

$$r_a=\frac{r(1-\sin\frac{1}{2}A)}{1+\sin\frac{1}{2}A}=\frac{r(\cos\frac{1}{4}A-\sin\frac{1}{4}A)^2}{(\cos\frac{1}{4}A+\sin\frac{1}{4}A)^2},$$

$$\text{and } \sqrt{(r_ar_b)}=\frac{r(\cos\frac{1}{4}A-\sin\frac{1}{4}A)(\cos\frac{1}{4}B-\sin\frac{1}{4}B)}{(\cos\frac{1}{4}A+\sin\frac{1}{4}A)(\cos\frac{1}{4}B+\sin\frac{1}{4}B)}$$

$$=\frac{r\cos\frac{A+\pi}{4}\cos\frac{B+\pi}{4}\cos\frac{C+\pi}{4}}{\cos\frac{A-\pi}{4}\cos\frac{B-\pi}{4}\cos\frac{C-\pi}{4}}=\frac{r(\cos\frac{1}{2}A+\cos\frac{1}{2}B-\cos\frac{1}{2}C)}{\cos\frac{1}{2}A+\cos\frac{1}{2}B+\cos\frac{1}{2}C}.$$

In like manner can be found $\sqrt{(r_br_c)}$ and $\sqrt{(r_cr_a)}$. Then by adding,

$$r=\sqrt{(r_ar_b)}+\sqrt{(r_br_c)}+\sqrt{(r_cr_a)}.$$

The length of I_2A is $R_a+r+\text{cosec}\frac{1}{2}A=R_a\text{cosec}\frac{1}{2}A$; thence

$$\frac{1}{R_a}=\frac{1-\sin\frac{1}{2}A}{r(1+\sin\frac{1}{2}A)}, \text{ as before,}$$

$$\frac{1}{\sqrt{(R_aR_b)}}=\frac{\cos\frac{1}{2}A+\cos\frac{1}{2}B-\cos\frac{1}{2}C}{r(\cos\frac{1}{2}A+\cos\frac{1}{2}B+\cos\frac{1}{2}C)}.$$

Similarly, can be found $\frac{1}{\sqrt{(R_bR_c)}}$ and $\frac{1}{\sqrt{(R_cR_a)}}$. Then by adding,

$$\frac{1}{r}=\frac{1}{\sqrt{(R_aR_b)}}+\frac{1}{\sqrt{(B_bR_c)}}+\frac{1}{\sqrt{(R_cR_a)}}.$$

II. Solution by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud. Gloucestershire, England.

It is clear that $r\text{cosec}\frac{1}{2}A+r+R_a=R_a\text{cosec}\frac{1}{2}A$.

$$\therefore \Sigma\frac{1}{\sqrt{(R_aR_b)}}=\frac{1}{r}\ \Sigma\sqrt{\frac{(1-\sin\frac{1}{2}A)(1--\sin\frac{1}{2}B)}{(1+\sin\frac{1}{2}A)(1+\sin\frac{1}{2}B)}}$$

$$=\frac{1}{r}\frac{\Sigma(\cos\frac{1}{4}A-\sin\frac{1}{4}A)(\cos\frac{1}{4}B-\sin\frac{1}{4}B)(\cos\frac{1}{4}C+\sin\frac{1}{4}C)}{\pi(\cos\frac{1}{4}A+\sin\frac{1}{4}A)(\cos\frac{1}{4}B+\sin\frac{1}{4}B)(\cos\frac{1}{4}C+\sin\frac{1}{4}C)}$$

$$=\frac{1}{r}\ \frac{\Sigma\cos\dfrac{A+\pi}{4}\cos\dfrac{B+\pi}{4}\cos\dfrac{C-\pi}{4}}{\pi\cos\dfrac{A-\pi}{4}}$$

$$=\frac{1}{r}\frac{\Sigma(\cos\frac{1}{2}A+\cos\frac{1}{2}B-\cos\frac{1}{2}C)}{\Sigma\cos\frac{1}{2}A}=\frac{1}{r}.$$

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.

162. Proposed by J. D. PALMER, Providence, Ky.

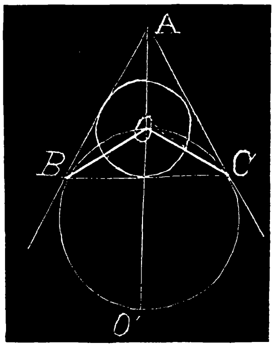
Given the distances from the vertices of a triangle, ABC, to the center of the circle, to construct the triangle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

AO, BO, $CO=a$, b, c, respectively, where O is the center of the in-circle; BC, AC, $AB=x$, y, z, respectively. Let O_1 be the center of the ex-circle opposite A. Then

$$AO^2=\frac{(p-x)^2}{\cos^2\frac{1}{2}A},\ \text{where}\ p=\tfrac{1}{2}(x+y+z).$$

$$\therefore AO^2=\left[\frac{p-x}{p}\right]yz=yz-\frac{xyz}{p}=yz-4Rr=a^2.$$



Similarly, $BO^2=\left[\dfrac{p-y}{p}\right]xz=xz-4Rr=b^2.$

$$CO^2=\left[\frac{p-z}{p}\right]xy=xy-4Rr=c^2.$$

$$\therefore AO.BO.CO=\frac{\triangle\ xyz}{p^2}=4R^2r^2\ \text{or}\ 2\sqrt{(abc)}=4Rr$$

$$\therefore a^2+2\sqrt{(abc)}=yz,\ b^2+2\sqrt{(abc)}=xz,$$

$$c^2=2\sqrt{(abc)}=xy:$$

$$\therefore x[a^2+2\sqrt{(abc)}=y[b^2+2\sqrt{(abc)}]=z[c^2+2\sqrt{(abc)}]$$